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*Published in:*  
Automatica

*DOI:*  
[10.1016/j.automatica.2008.03.016](https://doi.org/10.1016/j.automatica.2008.03.016)

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*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
2008

[Link to publication in University of Groningen/UMCG research database](#)

### *Citation for published version (APA):*

Mosca, E., Tesi, P., & Zhang, J. (2008). Feasibility of horizon-switching predictive control under positional and incremental input saturations. *Automatica*, 44(11), 2936-2939.  
<https://doi.org/10.1016/j.automatica.2008.03.016>

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## Technical communiqué

Feasibility of horizon-switching predictive control under positional and incremental input saturations<sup>☆</sup>Edoardo Mosca<sup>a,\*</sup>, Pietro Tesi<sup>a</sup>, Jingxin Zhang<sup>b</sup><sup>a</sup> Dipartimento di Sistemi e Informatica, Università di Firenze, Via di S. Marta, 3, 50139 Firenze, Italy<sup>b</sup> Department of Electrical and Computer Systems Engineering, Monash University, Clayton, Vic 3800, Australia

## ARTICLE INFO

## Article history:

Received 10 April 2007

Received in revised form

10 December 2007

Accepted 17 March 2008

Available online 9 October 2008

## Keywords:

Switching control

Predictive control

Control of input-saturated systems

Anti-windup control

Nonlinear control

## ABSTRACT

This paper proves the validity of a basic feasibility property which makes it possible to extend, to the case of set-point tracking in ANCBI systems under joint positional and incremental input saturations, the approach in [Mosca, E. (2005). Predictive switching supervisory control of persistently disturbed input-saturated plants. *Automatica*, 41, 55–67] restricted to positional input saturation and the pure regulation problem. An example on the use of the proposed technique is considered.

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## 1. Introduction

This paper studies the possible extension of the approach in Mosca (2005), which only concerns positional input saturations and the pure regulation problem, to the more general case of joint positional and incremental input saturation and set-point tracking problem. While positional input saturations have attracted a great deal of interest in the literature, fewer results apply to joint positional and incremental input saturations. The latter are a serious challenge in many automatic control applications, e.g. flight control (Dornheim, 1992; Lenorovitz, 1990). Joint constraints on both input magnitude and increments were considered in Trygve, Murray, and Fossen (1997) for the particular case of a plant consisting of a chain of cascade integrators. More generally, Lin (1995, 1997) showed that ANCBI (asymptotically null controllable with bounded inputs) systems are semi-globally stabilizable by linear feedback also in the presence of both constraints. For other contributions to the topic, see also Feng, Palaniswami, and Zhu (1992), Hanson and Stengel (1984), Lin, Pachter, Banda, and Shamash (1997), Tyan and Bernstein (1997).

However, apart from a few exceptions (e.g. see Angeli, Casavola, and Mosca (2000), Mhaskar and Kennedy (2008), Scokaert and Mayne (1998)), all these contributions deal mainly with the stabilization issue, and focus little attention on the performance of the overall controlled system. In Mosca (2005) the problem was reconsidered from the viewpoint of both stability and performance of systems subject to only positional input saturations.

## 2. Problem formulation

Consider the following discrete-time LTI ANCBI system

$$\begin{cases} x(t+1) = \Phi x(t) + Gu(t) + \xi \\ y(t) = Hx(t) + \zeta \end{cases} \quad (1)$$

with  $(\Phi, G)$  reachable where  $\Phi$  has all its eigenvalues of modulus less than or equal to one with arbitrary multiplicities;  $t \in \mathbb{Z}_{-1} := \{-1, 0, 1, \dots\}$ ; state  $x \in \mathbb{R}^n$ ; input  $u \in \mathbb{R}^m$ ; output  $y \in \mathbb{R}^m$ ;  $\xi$  and  $\zeta$  are constant disturbances. The plant input  $u(t)$  and its increments  $\delta u(t) := u(t) - u(t-1)$ ,  $\forall t \in \mathbb{Z}_+ := \{0, 1, \dots\}$  are subject to the following saturation constraints

$$u(t) \in \mathcal{U} := \{u \in \mathbb{R}^m : |u|_i < \bar{U}\}, \quad (2)$$

$$\delta u(t) \in \mathcal{D} := \{\delta u \in \mathbb{R}^m : |\delta u|_i < \bar{\Delta}\}, \quad (3)$$

where  $i \in \overline{m} := \{1, 2, \dots, m\}$ ,  $\bar{U}$ ,  $\bar{\Delta}$  positive extended reals, and  $|u|_i$  and  $|\delta u|_i$  denote the absolute value of the  $i$ -th component of  $u$  and, respectively,  $\delta u$ . It is known that ANCBI

<sup>☆</sup> This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Mayuresh V. Kothare under the direction of Editor André L. Tits.

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systems are the only input-constrained systems for which it makes sense to consider stability and boundedness for any arbitrary initial state/disturbances. The aim is to find a feedback control which asymptotically stabilizes (1) subject to (2) and (3) and possibly yields an asymptotic offset-free tracking. In this connection a classic approach is to enforce an “integral action” from  $\varepsilon := y - r$  to  $u$ ,  $r$  being the output reference. The design can be carried out by resorting to the so-called *incremental model* of (1)

$$\begin{cases} \chi(t+1) = \mathcal{A} \chi(t) + \mathcal{B} \delta u(t) \\ \varepsilon(t) = \mathcal{C} \chi(t) \end{cases} \quad (4)$$

where  $\chi(t) := [\delta x'(t) \ \varepsilon'(t-1)]'$ , the prime denotes transpose,  $\delta x(t) := x(t) - x(t-1)$ , and

$$\mathcal{A} = \begin{bmatrix} \Phi & 0 \\ H & I_m \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} G \\ 0_m \end{bmatrix}, \quad \mathcal{C} = [H \ I_m]. \quad (5)$$

It is well known that a linear state-feedback law  $\delta u(t) = \mathcal{F} \chi(t)$ , which stabilizes (4), yields an offset-free steady-state tracking error. A necessary and sufficient condition for the existence of such a stabilizing linear state-feedback is as follows (Davison, 1976)

$$\det \begin{bmatrix} I_n - \Phi & G \\ H & 0_m \end{bmatrix} \neq 0. \quad (6)$$

Let  $\chi$  be the state of (4) at time 0, and  $\Omega_h(\chi)$  the set of all control increments  $\omega$  of length  $h$ ,  $\omega = [\delta u'(0), \dots, \delta u'(h-1)]'$ , which drive the system state to the zero-state  $0_\chi$  in  $h$  time-steps. Denote with  $\nu$  the reachability index of  $(\mathcal{A}, \mathcal{B})$  and let  $\delta u_h(\chi)$  be the element in  $\Omega_h(\chi)$  of minimum energy  $\omega' \omega$ . For  $h \geq \nu$ ,  $\delta u_h(\chi)$  is as follows

$$\begin{aligned} \delta u_h(\chi) &:= [\delta u'_h(0|\chi), \dots, \delta u'_h(h-1|\chi)]' \\ &= [\mathcal{F}'_h(0) \cdots \mathcal{F}'_h(h-1)]' \chi =: \mathcal{F}_h \chi, \end{aligned} \quad (7)$$

$$\mathcal{F}_h := -\mathcal{R}'_h \mathcal{G}_h^{-1} \mathcal{A}^h \in \mathbf{R}^{m \times n}, \quad (8)$$

where  $\mathcal{R}_h$  is the  $h$ -order reachability matrix  $\mathcal{R}_h := [\mathcal{A}^{h-1} \mathcal{B} \ \dots \ \mathcal{A} \mathcal{B} \ \mathcal{B}]$  and  $\mathcal{G}_h$  the  $h$ -order reachability Gramian  $\mathcal{G}_h := \mathcal{R}_h \mathcal{R}'_h$ . The integer  $h$  will be referred to as the *control horizon*. Note that  $\mathcal{F}_h(k)$  in (7) is given in terms of  $\mathcal{F}_h$  as follows

$$\mathcal{F}_h(k) = \begin{bmatrix} 0_{m \times m k} & I_m & 0_{m \times m(h-1-k)} \end{bmatrix} \mathcal{F}_h \in \mathbf{R}^{m \times n}. \quad (9)$$

Let

$$M_h(\chi) := \max \left\{ \frac{|\delta u_h(k|\chi)|_i}{\bar{\Delta}}; k+1 \in \overleftarrow{h}; i \in \overleftarrow{m} \right\}, \quad (10)$$

where  $[\delta u]_i$  denotes the  $i$ -th component of the vector  $\delta u$ . Note that the whole sequence  $\delta u_h(\chi)$  does not violate (3) if and only if  $M_h(\chi) < 1$ . As (4) is ANCB, it is always possible to find a large enough horizon  $h$  so as to satisfy  $M_h(\chi) < 1$ . In fact, it can be shown (Mosca, 2005) that for an ANCB system

$$M_h(\chi) \leq \bar{M} h^{-1} \|\chi\|, \quad (11)$$

where  $\bar{M}$  is a positive real depending on  $(\mathcal{A}, \mathcal{B})$ . If only input-increment saturations are present, at a generic time  $t$ ,  $h(t)$  can be chosen, according to a suitable logic, such that  $M_{h(t)}(\chi(t)) < 1$  and the input increment to (4) can be set as  $\delta u(t) = F_{h(t)} \chi(t)$ . Here,  $F_{h(t)} = \mathcal{F}_{h(t)}(0)$  is recognized to be the feedback-gain matrix of the receding horizon regulation related to the zero-terminal state minimum energy control problem of horizon  $h(t)$ .

### 3. Feasibility under incremental and positional input saturations

Let for  $k+1 \in \overleftarrow{h}$ ,  $i \in \overleftarrow{m}$ ,

$$\mathcal{M}_h(\chi) := \max \left\{ \tilde{\alpha} \frac{|\delta u_h(k|\chi)|_i}{\bar{\Delta}}, \alpha \frac{|u_h(k|\chi)|_i}{\bar{U}} \right\}, \quad (12)$$

where  $(\tilde{\alpha} = 1, \alpha = 0)$  corresponds to only incremental saturations;  $(\tilde{\alpha} = 0, \alpha = 1)$  pertains to only positional saturations;  $(\tilde{\alpha} = 1, \alpha = 1)$  to joint incremental and positional saturations. The fundamental question for extending (Mosca, 2005) to the present case is whether, given an arbitrary  $\chi$ , there exist  $h$  such that  $\mathcal{M}_h(\chi) < 1$  for any of the possible pair  $(\tilde{\alpha}, \alpha)$ . If this is the case, one can always find a (virtual) input increment sequence (7) of large enough length  $h$  for which the saturation constraints (2) and (3) are jointly satisfied.

Consider the orthogonal decomposition

$$\mathbf{R}^N = \mathcal{R}((\mathcal{A}^N)') \oplus \mathcal{N}(\mathcal{A}^N), \quad (13)$$

where  $\mathcal{N}(\mathcal{A}^N) = \mathcal{N}(\mathcal{A}^h)$ ,  $\forall h \geq N = \dim(\mathcal{A})$ , and  $\mathcal{R}(\cdot)$  and  $\mathcal{N}(\cdot)$  denote range-space and, respectively, null-space. As, if  $\chi^\perp \in \mathcal{N}(\mathcal{A}^N)$ ,  $\delta u_h(k|\chi^\perp) = 0$ ,  $\forall h \geq N$ , we can restrict the study to states in  $\mathcal{R}((\mathcal{A}^N)')$ . This amounts to assuming w.l.o.g.  $\mathcal{A}$  non singular. Under such an assumption, the following properties hold ( $O(h^{-1})$  stands for a quantity of the order of  $h^{-1}$  or which vanishes at a faster rate as  $h \rightarrow \infty$ ).

**Lemma 1.** Consider the incremental ANCB model (4)–(6). Let  $F_h = \mathcal{F}_h(0)$ , with  $\mathcal{F}_h$  and  $\mathcal{F}_h(k)$  as in (8) and, respectively, (9). Then, the following properties hold,  $\forall k+1 \in \overleftarrow{h}$ ,

$$\mathcal{F}_{h+1}(k) = \mathcal{F}_h(k) [I + O(h^{-1})]. \quad (14)$$

If  $\mathcal{A}_h := \mathcal{A} + \mathcal{B} F_h$  and  $l = [0' \ \varepsilon']' \in \mathbf{R}^N$ ,

$$\mathcal{F}_h(k+1) = \mathcal{F}_h(k) \mathcal{A}_h [I + O(h^{-1})], \quad (15)$$

$$\mathcal{F}_h(k+1)l = \mathcal{F}_h(k) [I + O(h^{-1})] l. \quad (16)$$

**Proof.** See the Appendix.  $\square$

In order to compute  $\delta u_h(k|\chi)$  for large  $h$ , let  $\chi = \chi(0) := l + v$  where  $l := [0' \ \varepsilon'(-1)]'$ ,  $v := [\delta x'(0) \ 0']'$ . Then, by linearity of  $\delta u_h(k|\cdot)$  one has  $\delta u_h(k|\chi) = \delta u_h(k|l) + \delta u_h(k|v)$ . Further,

$$\begin{aligned} \delta u_h(k|l) &= \mathcal{F}_h(k)l = \mathcal{F}_h(k-1) \mathcal{A}_h [I + O(h^{-1})] l \\ &= \mathcal{F}_h(k-1) [I + O(h^{-1})] l \\ &= F_h [I + O(h^{-1})] l, \end{aligned} \quad (17)$$

where the second equality follows from (15), and the third from the fact that  $\mathcal{A}_h l = l + O(h^{-1})$ . Consequently,

$$\begin{aligned} u_h(k|\chi) &= u(-1) + \sum_{i=0}^k \delta u_h(i|\chi) \\ &= u(-1) + (k+1)F_h l + u_h(k|v) + O(h^{-1}), \end{aligned} \quad (18)$$

where  $O(h^{-1})$ , the rightmost term in (18), arises by taking into account that  $F_h = O(h^{-1})$ , and consequently  $\sum_{i=0}^k F_h O(h^{-1})l = O(h^{-1})$ ,  $k+1 \in \overleftarrow{h}$ .

Now, one must have

$$u_h(h-1|\chi) = u^\infty, \quad (19)$$

if  $u^\infty$  denotes the input vector to (1) which in steady-state yields the desired set-point  $r$  at the output of (1). Using (19) in (18), one

finds  $F_h l = [u^\infty - u(-1)] h^{-1} - u_h(h-1|v) h^{-1} + O(h^{-2})$ . Therefore,  $k+1 \in \overleftarrow{h}$ ,

$$u_h(k|\chi) = u(-1) + \frac{k+1}{h} [u^\infty - u(-1)] + u_h(k|v) - \frac{k+1}{h} u_h(h-1|v) + O(h^{-1}). \quad (20)$$

We now turn to show that  $\delta u_h(k|v) = O(h^{-1})$  and, similarly,  $u_h(k|v) = O(h^{-1})$ . In fact,

$$\begin{aligned} \delta u_h(k|v) &= \mathcal{F}_h(k)v = \mathcal{F}_h(k-1)\mathcal{A}_h[I + O(h^{-1})]v \\ &= F_h \mathcal{A}_h^k [I + O(h^{-1})]v, \end{aligned} \quad (21)$$

where the second equality follows from (15). That  $\delta u_h(k|v) = O(h^{-1})$  follows from the fact that  $F_h = O(h^{-1})$  and that  $\mathcal{A}_h$  is a stability matrix.

Using these two properties, it is easy to see that also  $u_h(k|v) = O(h^{-1})$ .

Summing up,  $k+1 \in \overleftarrow{h}$ ,

$$\delta u_h(k|\chi) = F_h \mathcal{A}_h^k \chi + O(h^{-2}), \quad (22)$$

$$u_h(k|\chi) = u(-1) + \frac{k+1}{h} [u^\infty - u(-1)] + O(h^{-1}). \quad (23)$$

Eqs. (22) and (23) show that for any initial state  $\chi \in \mathbf{R}^N$  it is always possible to find a large enough control horizon  $h$  so as to make the virtual input increments  $\delta u_h(\cdot|\chi)$  and virtual inputs  $u_h(\cdot|\chi)$  compatible with constraints (2) and (3) provided that  $u(-1), u^\infty \in \mathcal{U}$ . Notice that the latter property amounts to assuming that  $\xi, \zeta$  and  $r$  are jointly within the input control range. Properties (22) and (23) are summed up in the following

**Feasibility Property.** Consider the reachable ANCB system (1) subject to joint input positional and incremental saturation constraints (2) and (3). Then, for every  $\chi \in \mathbf{R}^N$  and in the presence of constant disturbances  $\xi, \zeta$  and set-point  $r$  for which  $u^\infty \in \mathcal{U}$ , control horizons  $h$  can always be found so that  $u_h(\chi) \in \mathcal{U}$  and  $\delta u_h(\chi) \in \mathcal{D}$  provided that  $u(-1) \in \mathcal{U}$ .

Taking into account the Feasibility Property, one can adopt as a switching logic for choosing  $h$  at each time  $t$ , call it  $h(t)$ , a natural extension of the one as in Mosca (2005) so as to obtain a closed-loop switched system enjoying offset-free asymptotic tracking under joint incremental and positional input saturations. Specifically,  $\delta u_{h(t)} = \mathcal{F}_{h(t)} \chi(t)$  is chosen according to the following hysteresis switching logic ( $\underline{h} \geq N$ )

$$h(t) = \begin{cases} \tilde{h}(t), & \text{if } M_{\tilde{h}(t)}(\chi(t)) \leq 1 \\ \hat{h}(t), & \text{otherwise.} \end{cases} \quad (24)$$

$$\tilde{h}(t) := \max\{\underline{h}, h(t-1) - 1\}$$

$$\hat{h}(t) := \min\{h \in \mathbf{Z}_+ : h \geq h(t-1); M_h(\chi(t)) \leq 1 - \epsilon\}.$$

In (24),  $\underline{h}$  denotes the minimum horizon whose choice is up to the designer (roughly, the larger  $\underline{h}$ , the narrower the frequency bandwidth of the closed-loop system in steady-state). As proved in Mosca (2005), stability of the switched system is ensured by the crucial condition  $h(t) \geq h(t-1) - 1$ . In words, the horizon is not allowed to decrease more than one unit in a single time-step, while arbitrary increases of the horizon do not destroy stability. An extension of the present approach to persistent time-varying disturbances, as in Mosca (2005), is currently under development.

**Remark 1.** There is a feature of the feedback-gains (8) which can be conveniently exploited for checking the condition  $\mathcal{M}_h(\chi) < 1$ . Let  $\chi_{\mathcal{A}}(z)$  be the characteristic polynomial of  $\mathcal{A}$ ,  $\chi_{\mathcal{A}}(z) := \det(zI - \mathcal{A}) = z^N + a_1 z^{N-1} + \dots + a_{N-\rho} z^\rho$ , where  $a_{N-\rho} \neq 0$ , and  $\rho$  denotes

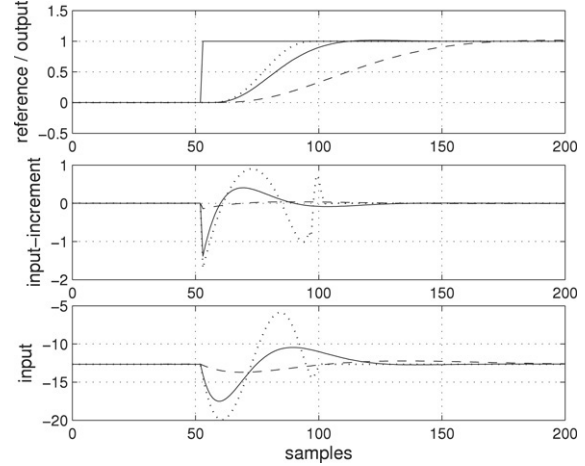


Fig. 1. Response to step reference: Dotted line,  $h = 5$ ; Solid line,  $h = 50$ ; Dashed line,  $h = 100$ .

the number of the zero roots of  $\chi_{\mathcal{A}}(z)$ . By the Cayley–Hamilton theorem (Brockett, 1970), it follows that  $\chi_{\mathcal{A}}(\mathcal{A}) = \mathcal{A}^N + \dots + a_{N-\rho} \mathcal{A}^\rho = 0$ . Therefore, by the form of the feedback-gains,

$$\mathcal{F}_h(k) = c_1 \mathcal{F}_h(k-1) + \dots + c_{N-\rho} \mathcal{F}_h(k-N+\rho), \quad (25)$$

for  $N-\rho \leq k \leq h-1$ , and  $c_i = -a_{N-\rho-1}/a_{N-\rho}$ ,  $i \in \overleftarrow{N-\rho}$ , with  $a_0 = 1$ . Hence, as  $\delta u_h(k|\chi) = \mathcal{F}_h(k)\chi$ ,

$$\begin{aligned} \delta u_h(k|\chi) &= c_1 \delta u_h(k-1|\chi) + \dots + c_{N-\rho} \delta u_h(k-N+\rho|\chi). \end{aligned} \quad (26)$$

Eqs. (25) and (26) imply that, in order to perform the admissibility test  $\mathcal{M}_h(\chi) \leq 1$  for the virtual incremental and positional input sequences, it suffices to store the first  $N-\rho$  feedback-gains  $\mathcal{F}_h(k)$ ,  $k+1 \in \overleftarrow{N-\rho}$ , compute the first  $N-\rho$  input increments  $\delta u_h(k|\chi) = \mathcal{F}_h(k)\chi$ ,  $k+1 \in \overleftarrow{N-\rho}$ , while all the remaining ones in the sequence can be generated via the recursions (26).

#### 4. An example

Consider the control of the roll angle of an aircraft (Vegte, 1994). The discrete-time positional system (zero-order hold and 5 ms sampling time) is as follows

$$\begin{aligned} x(t+1) &= \begin{bmatrix} 0.9956 & 0.0177 & 0.0004 \\ 0 & 0.7788 & 0.0354 \\ 0 & 0 & 1 \end{bmatrix} x(t) \\ &+ \begin{bmatrix} 0.0000 \\ 0.0007 \\ 0.0395 \end{bmatrix} u(t) + \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \xi_3(t) \end{bmatrix} \end{aligned}$$

$$y(t) = [-3.9528 \quad 0 \quad 0] x(t) + \zeta(t). \quad (27)$$

System responses to a step reference, for different values of  $h$ , are shown in Fig. 1. Here, the qualitative behavior of (22) and (23) as a function of  $h$  is confirmed in that, as  $h$  increases,  $\delta u_h$  tends to a constant, while  $u_h$  tends to a straight line connecting the initial and required final value of  $u$ . Next, we consider the Feasibility Property, which states that the switched system is globally asymptotically stabilized and achieves offset-free set-point tracking for the class of disturbance and the reference sequences which become constant in a finite time (this encompasses, from a practical viewpoint, the case of infrequent set-point changes). Consider the problem of stabilizing (27) and making  $y$  to track a reference  $r$ ,

$$r(t) := \begin{cases} 3, & 0 \leq t < 300 \\ 0, & 300 \leq t < 600 \\ -3, & t \geq 600 \end{cases} \quad (28)$$

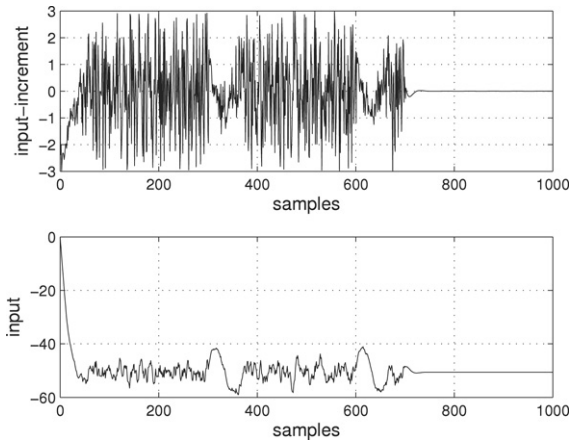


Fig. 2. (a) Input-increment  $\delta u$ ; (b) Input  $u$ .

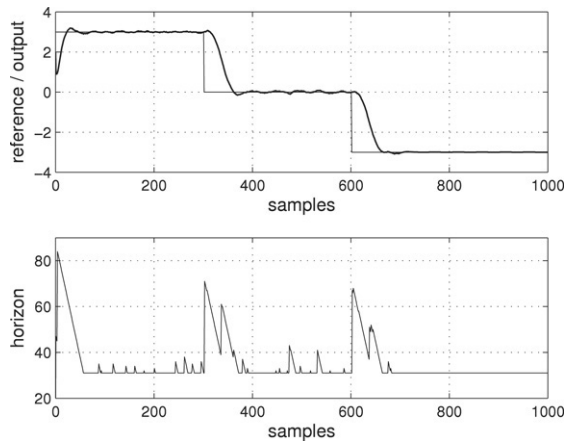


Fig. 3. (a) Reference  $r$  and output  $y$ ; (b) Control horizon  $h$ ,  $\bar{h} = 30$ .

using a control action  $\delta u = F_h(t)\chi(t)$ ,  $\underline{h} = 30$ , which saturates outside  $[-3, 3]$ . According to Lemma 1,  $F_h = \mathcal{F}_h(0)$ , with  $\mathcal{F}_h$  and  $\mathcal{F}_h(k)$  as in (8), respectively, (9). The control input  $u(t)$  applied to (27) saturates outside  $[-60, 60]$ . The simulations in Figs. 2 and 3 refer to disturbances uniformly distributed,  $\xi_1 \in [0 \pm 0.003]$ ,  $\xi_2 \in [1 \pm 0.05]$ ,  $\xi_3 \in [2 \pm 10^{-5}]$  and  $\zeta \in [-1 \pm 0.01]$ , which become constant after  $t \geq 700$ . Notice that the constraints on the control law are compatible with the set-point tracking problem, as the steady-input remains, as shown in Fig. 2(b), in a neighborhood of the input  $u^\infty = -50.5964$ , corresponding to constant disturbances. Fig. 3(b) shows what can be called the horizon resetting property of the algorithm. Starting at sample time  $t_0$ , the switching logic selects the minimum control horizon  $h(t_0)$ , capable of satisfying the saturation constraints. Then, in the case of constant set-points and disturbances, the horizon decreases from  $h(t_0)$  by one unit at each time-step, up to  $\underline{h}$ . Thereafter, for  $t \geq h(t_0) - \underline{h}$ ,  $h(t)$  equals  $\underline{h}$ . Otherwise, in the presence of set-point or disturbance changes, the horizon is re-selected, at any time, in accordance with the new steady-state control.

## 5. Conclusions

The main result of this paper is the statement referred to as the *Feasibility Property*. It allows one to consider possible extensions of the approach in Mosca (2005) to the tracking problem of systems under joint incremental and positional input saturations. A simulation example illustrates the effectiveness of the technique proposed.

## Appendix

**Proof of Lemma 1.** From (7) one finds

$$\begin{aligned}\mathcal{F}_{h+1}(k) &= -\mathcal{B}'(\mathcal{A}^{h-k})'\mathcal{G}_{h+1}^{-1}\mathcal{A}^{h+1} \\ &= -\mathcal{B}'(\mathcal{A}^{h-1-k})'(\mathcal{A}'\mathcal{G}_{h+1}^{-1}\mathcal{A})\mathcal{A}^h.\end{aligned}\quad (29)$$

Now

$$\begin{aligned}(\mathcal{A}'\mathcal{G}_{h+1}^{-1}\mathcal{A})^{-1} &= \mathcal{A}^{-1}\mathcal{G}_{h+1}\mathcal{A}^{-T} = \mathcal{G}_h + \mathcal{A}^{-1}\mathcal{B}\mathcal{B}'\mathcal{A}^{-T} \\ &= \mathcal{G}_h(I + \mathcal{G}_h^{-1}\mathcal{A}^{-1}\mathcal{B}\mathcal{B}'\mathcal{A}^{-T}).\end{aligned}\quad (30)$$

Therefore

$$\begin{aligned}\mathcal{A}'\mathcal{G}_{h+1}^{-1}\mathcal{A} &= (I + \mathcal{G}_h^{-1}\mathcal{A}^{-1}\mathcal{B}\mathcal{B}'\mathcal{A}^{-T})^{-1}\mathcal{G}_h^{-1} \\ &= \mathcal{G}_h^{-1}(I + O(h^{-1})),\end{aligned}\quad (31)$$

where, as shown in Mosca (2005),  $\mathcal{G}_h^{-1} = O(h^{-1})$ . Hence, (14) follows. The first equality in the next equation was shown (Mosca, 2005) to hold for the feedback-gains in (7), provided that  $\mathcal{A}_h := \mathcal{A} + \mathcal{B}F_h$ ,

$$\mathcal{F}_h(k+1) = \mathcal{F}_{h-1}(k)\mathcal{A}_h = \mathcal{F}_h(k)\mathcal{A}_h[I + O(h^{-1})] \quad (32)$$

where the last equality follows from (14). By (15), the first equality in the following equation holds

$$\begin{aligned}\mathcal{F}_h(k+1)l &= \mathcal{F}_h(k)\mathcal{A}_h[I + O(h^{-1})]l \\ &= \mathcal{F}_h(k)[\mathcal{A} + \mathcal{B}F_h][I + O(h^{-1})]l \\ &= \mathcal{F}_h(k)[I + O(h^{-1})]l,\end{aligned}\quad (33)$$

where the last equality holds because, as can be checked,  $\mathcal{A}l = l$ , and, by (11),  $F_h = O(h^{-1})$ .  $\square$

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